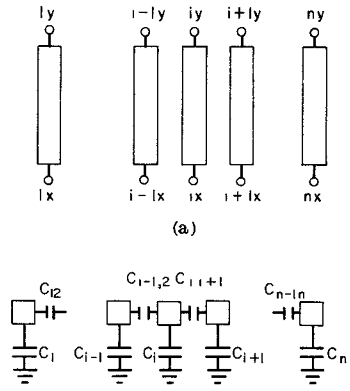

Fig. 1. (a) n coupled transmission lines. (b) Capacitances per unit length of the lines of Fig. 1(a).

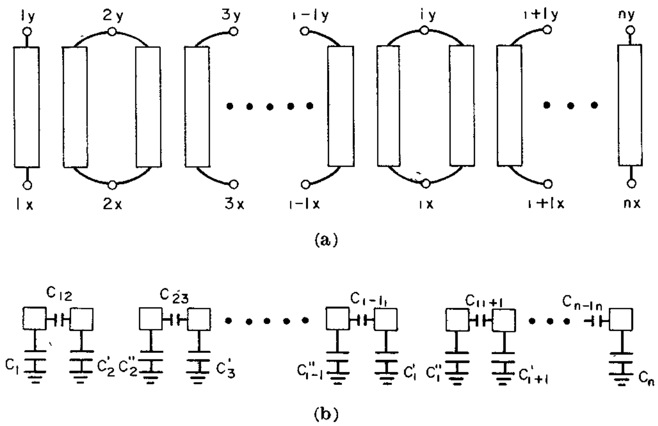

Fig. 2. (a) Network consisting of $n-1$ pairs of coupled lines. (b) Capacitances per unit length of the lines of Fig. 2(a).

$$
\begin{aligned}
I_{i+1,a}{}^1 &= -jV_1 \cot(\theta) Y_{01}{}^{i+1}/2 & I_{i+1,a}{}^2 &= jV_1 \cot(\theta) Y_{02}{}^{i+1}/2 \\
I_{i+1,b}{}^1 &= jV_1 Y_{01}{}^{i+1}/(2 \sin(\theta)) & I_{i+1,b}{}^2 &= -jV_1 Y_{02}{}^{i+1}/(2 \sin(\theta)) \\
I_{i-1,a}{}^1 &= -jV_1 \cot(\theta) Y_{01}{}^{i-1}/2 & I_{i-1,a}{}^2 &= jV_1 \cot(\theta) Y_{02}{}^{i-1}/2 \\
I_{i-1,b}{}^1 &= jV_1 Y_{01}{}^{i-1}/(2 \sin(\theta)) & I_{i-1,b}{}^2 &= -jV_1 Y_{02}{}^{i-1}/(2 \sin(\theta))
\end{aligned}
\tag{4}
$$

where $I_{j,v}{}^1$ is the current in the jv node for mode 1 excitation and similarly $I_{j,v}{}^2$ is the current in the jv node for mode 2 excitation. Equation (2) then yields, when $p = -j \cot(\theta)$ and $t = \sec(\theta)$,

$$
\begin{aligned}
y_{i,i}{}^{a,a} &= -j \cot(\theta)(Y_{01}{}^i + Y_{02}{}^i)/2 = \nu(C_i + C_{i-1,i} + C_{i,i+1})p \\
y_{ii}{}^{a,b} &= j(Y_{01}{}^i + Y_{02}{}^i)/(2 \sin(\theta)) = -\nu(C_i + C_{i-1,i} + C_{i,i+1})pt \\
y_{i,i+1}{}^{a,a} &= j \cot(\theta)(Y_{02}{}^{i+1} - Y_{01}{}^{i+1})/2 = -\nu(C_{i,i+1})p \\
y_{i,i+1}{}^{a,b} &= -j(Y_{02}{}^{i+1} - Y_{01}{}^{i+1})/(2 \sin(\theta)) = \nu(C_{i,i+1})pt \\
y_{i,i-1}{}^{a,a} &= j \cot(\theta)(Y_{02}{}^{i-1} - Y_{01}{}^{i-1})/2 = -\nu(C_{i-1,i})p \\
y_{i,i-1}{}^{a,b} &= -j(Y_{02}{}^{i-1} - Y_{01}{}^{i-1})/(2 \sin(\theta)) = \nu(C_{i-1,i})pt.
\end{aligned}
\tag{5}
$$

We see from (3) that if $|i - j|$ is greater than 1 that $Y_{01}{}^j = Y_{02}{}^j$ and hence superposition of mode 1 and mode 2 excitation yields zero current, and $y_{i,j}{}^{u,v} = 0$. We find using (5), reciprocity and the symmetry of the network, that the admittance matrix is that of (1) where

$$
\begin{aligned}
Y_{11} &= \nu(C_1 + C_{12}) \\
Y_{ii} &= \nu(C_{i-1,i} + C_i + C_{i,i+1}) \\
Y_{nn} &= \nu(C_n + C_{n-1,n}) \\
Y_{i,i+1} &= -\nu C_{i,i+1}.
\end{aligned}
\tag{6}
$$

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A Useful Identity for the Analysis of a Class of Coupled Transmission-Line Structures

A. I. GRAYZEL

***Abstract*—In this letter an identity is proven which allows easy analysis of many coupled transmission-line structures.**

In this letter we will prove the following theorem: if only nearest neighbor couplings are considered n commensurate-coupled transmission lines can be reduced to a network consisting of $n - 1$ pairs of coupled lines.

To prove this theorem we will prove that the network of Fig. 1 and the network of Fig. 2 are equivalent. In Fig. 1(a) is shown a

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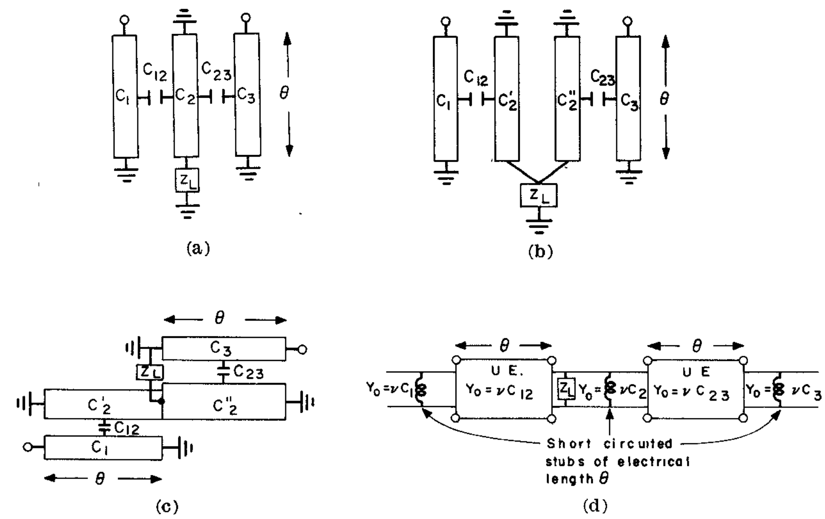

Fig. 3. (a) Loaded interdigital filter. Shunt and mutual capacitances per unit length as indicated. (b) Equivalent circuit of Fig. 3(a) consisting of two pairs of coupled lines. (c) Network equivalent to the network of Fig. 3(b). (d) Network equivalent to the network of Fig. 3(c).

network consisting of n coupled lines. The capacitances per unit length between coupled lines and shunt capacitances to ground are shown in Fig. 1(b). In Fig. 2(a) is shown a network consisting of $n-1$ pairs of coupled lines. The capacitances per unit length between the pairs of coupled lines and shunt capacitances to ground are shown in Fig. 2(b). These pairs of coupled lines are connected to $2n$ nodes as shown in Fig. 2(a). We will now show that the network of Fig. 1 and that of Fig. 2 are exactly equivalent when

$$C_i' + C_i'' = C_i, \qquad i = 2,3,\cdots,n-1. \tag{1}$$

(It should be noted that C_1, C_n, and $C_{i\ i+1}$ are the same for both networks.)

We shall prove this identity by showing that the $2n \times 2n$ admittance matrix of the network of Fig. 1 is identical to that of Fig. 2. Let us define the current, voltage, and admittance matrices by the matrix equation

$$\begin{bmatrix} I_{1x} \\ I_{1y} \\ I_{2x} \\ \vdots \\ I_{ny} \end{bmatrix} = [y_{ij}^{u,v}] \begin{bmatrix} v_{1x} \\ v_{1y} \\ v_{2x} \\ \vdots \\ v_{ny} \end{bmatrix}. \tag{2}$$

I_{jv} is the current flowing into node jv, where j ranges from 1 to n and v is equal to either x or y. Similarly, V_{iu} is the voltage between node iu and the ground planes.

It is shown in [2] and [6] that the admittance matrix of the network of Fig. 1 is given by

where (see [6])

$$\begin{aligned} y_{11} &= \nu(C_1 + C_{12}) \\ y_{ii} &= \nu(C_{i,i-1} + C_i + C_{i,i+1}), \quad i = 2,3,\cdots,n-1 \\ y_{nn} &= \nu(C_{n-1,n} + C_n) \\ y_{i,i+1} &= -\nu C_{i,i+1}, \qquad i = 1,2,\cdots,n-1 \\ p &= -j\cot\theta \\ t &= \sec\theta. \end{aligned} \tag{4}$$

It is shown in the Appendix that the admittance matrix of Fig. 2 is also given by (3) when the condition of (1) is satisfied. This completes the proof.

This identity is extremely useful for analyzing many interdigital structures. Consider the network of Fig. 3(a). Using the previous identity we can redraw the network as shown in Fig. 3(b) and (c). Using the equivalent circuits given in [3] we see that this network is equivalent to the network of Fig. 3(d). All of the equivalent circuits given in [4] can be derived by similar techniques and are immediately obvious. Further, given any group of coupled lines with one of the interior lines short circuited, such as shown in Fig. 4(a), one can always divide the group into two groups of coupled lines as shown in Fig. 4(b).

APPENDIX

The four-by-four admittance matrix of two coupled lines is given by (3) when n is set equal to 2. The matrix is given by

$$[y_{ij}^{uv}] = p \begin{bmatrix} y_{11} & -y_{11}t & y_{12} & -y_{12}t & 0 & 0 & \cdot & \cdot & \cdot & \cdots & \cdot & 0 & 0 \\ -y_{11}t & y_{11} & -y_{12}t & y_{12} & 0 & 0 & \cdot & \cdot & \cdot & \cdots & \cdot & 0 & 0 \\ y_{12} & -y_{12}t & y_{22} & -y_{22}t & y_{23} & -y_{23}t & 0 & 0 & \cdot & \cdots & \cdot & 0 & 0 \\ -y_{12}t & y_{12} & -y_{22}t & y_{22} & -y_{23}t & y_{23} & 0 & 0 & \cdot & \cdots & \cdot & 0 & 0 \\ 0 & 0 & y_{23} & -y_{23}t & y_{33} & -y_{33}t & y_{34} & -y_{34}t & 0 & \cdots & \cdot & 0 & 0 \\ 0 & 0 & -y_{23}t & y_{23} & -y_{33}t & y_{33} & -y_{34}t & y_{34} & 0 & \cdots & \cdot & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & y_{nn} & -y_{nn}t \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & -y_{nn}t & y_{nn} \end{bmatrix} \tag{3}$$

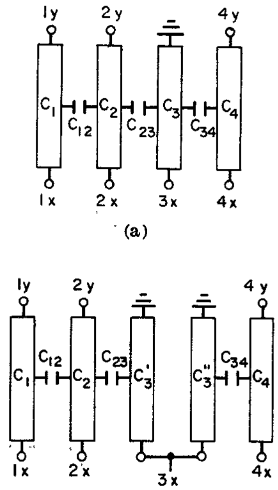

Fig. 4. (a) Interdigital structure with shunt and mutual capacitances per unit length as indicated. (b) Network equivalent to the network of Fig. 4(a) consisting of two groups of coupled lines.

$$\begin{bmatrix} y_{11} & -y_{11}t & y_{12} & -y_{12}t \\ -y_{11}t & y_{11} & -y_{12}t & y_{12} \\ y_{12} & -y_{12}t & y_{22} & -y_{22}t \\ -y_{12}t & y_{12} & -y_{22}t & y_{22} \end{bmatrix} \qquad (5a)$$

where

$$y_{11} = \nu(C_1 + C_{12})$$
$$y_{22} = \nu(C_2 + C_{12})$$
$$y_{12} = -\nu C_{12}. \qquad (5b)$$

Let us consider now the circuit of Fig. 5. Here we have two coupled lines connected to nodes ix, iy, $i + 1x$, and $i + 1y$ as shown in Fig. 5. The capacitances per unit length in shunt to ground and the mutual capacitance per unit length between the lines are C_i'', C_{i+1}', and $C_{i,i+1}$. There are no connections made to any of the other nodes. The admittance matrix of this network is given by (6)

$$[M_{i,i+1}] = p \begin{bmatrix} 0 & \cdots & 0 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 \\ \vdots & & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & & \vdots \\ 0 & \cdots & 0 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 \\ 0 & \cdots & 0 & y_{ii}'' & -y_{ii}''t & y_{i,i+1} & -y_{i,i+1}t & 0 & \cdots & 0 \\ 0 & \cdots & 0 & -y_{ii}''t & y_{ii}'' & -y_{i,i+1}t & y_{i,i+1} & 0 & \cdots & 0 \\ 0 & \cdots & 0 & y_{i,i+1} & -y_{i,i+1}t & y_{i+1,i+1}' & -y_{i+1,i+1}'t & 0 & \cdots & 0 \\ 0 & \cdots & 0 & -y_{i,i+1}t & y_{i,i+1} & -y_{i+1,i+1}t & y_{i+1,i+1}' & 0 & \cdots & 0 \\ 0 & \cdots & 0 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 \\ \vdots & & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & & \vdots \\ 0 & \cdots & 0 & 0 & 0 & 0 & 0 & 0 & & 0 \end{bmatrix} \qquad (6)$$

where

$$y_{ii}'' = \nu(C_i'' + C_{i,i+1})$$
$$y_{i+1,i+1}' = \nu(C_{i+1}' + C_{i,i+1})$$
$$y_{i,i+1} = -\nu C_{i,i+1}. \qquad (7)$$

The admittance matrix of the circuit of Fig. 2 is just the summation of $n - 1$ such matrices (6) and is therefore given by $\Sigma_{i=1}^{n-1} M_{i,i+1}$. By successive addition of the matrices $[M_{1,2}]$, $[M_{2,3}]$ to $[M_{n-1,n}]$ making use of the relationship

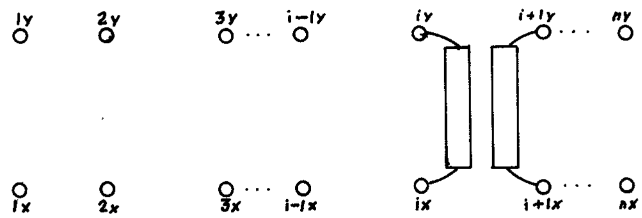

Fig. 5. Two coupled lines connected to four nodes of a field of $2n$ nodes.

$$y_{11}'' = y_{11} \qquad y_{nn}' = y_{nn}$$
$$y_{ii}' + y_{ii}'' = \nu(C_i' + C_{i-1,i} + C_i'' + C_{i,i+1}) = y_{ii} \qquad (8)$$

one obtains the matrix given in (3).

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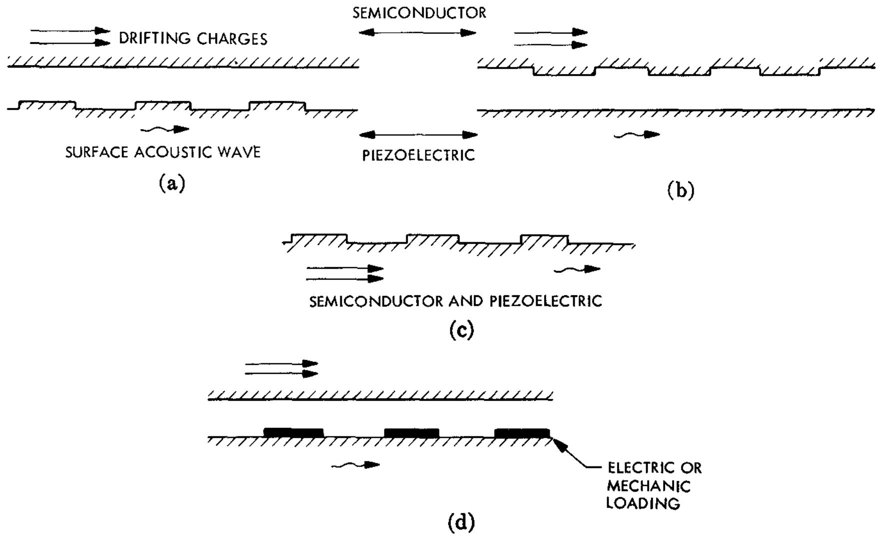

Fig. 1. Different schemes for a DFB surface acoustic wave oscillator.

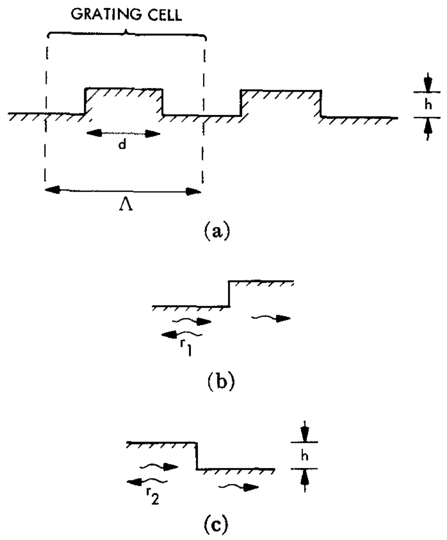

Fig. 2. Grating cell. r_1 is the reflection from a vertical surface elevation and r_2 is the reflection from the vertical surface depression.

Distributed Feedback Acoustic Surface Wave Oscillator

CHARLES ELACHI

***Abstract*—The application of the distributed feedback concept to generate acoustic surface waves is discussed. It is shown that surface corrugation of the piezoelectric boundary in a semiconductor-piezoelectric surface acoustic wave amplifier could lead to self-sustained oscillations.**

I. INTRODUCTION

The distributed feedback (DFB) concept has been recently used in the development of thin-film lasers [1]–[3], and its characteristics were the subject of many publications [4]–[7]. The basic idea is to replace the reflecting mirrors at the end of an amplifying medium by a Bragg grating throughout the medium which would generate a distributed feedback. In Fig. 1 we show a number of possible configurations which can be used for acoustic surface wave generation by having distributed feedback in a piezoelectric-semiconductor or acoustic surface wave amplifier. The distributed Bragg grating could consist of surface corrugation or periodic perturbation of any parameter which would affect the acoustic wave, electrostatic wave, or drifting charges. In this letter, we will use a simple model to evaluate the feasibility of a DFB surface acoustic wave oscillator using the scheme in Fig. 1(a).

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II. COUPLING COEFFICIENT

The feedback efficiency is expressed by the coupling coefficient between a forward and a backward wave. Let us consider a surface wave, of wavelength λ, propagating on a corrugated surface [Fig. 2(a)] where $h \ll \lambda$ and $\Lambda = \lambda/2$ (i.e., Bragg condition). Let r_1 be the reflection coefficient when the wave encounters a vertical surface elevation [Fig. 2(b)] and r_2 the reflection coefficient at a vertical surface depression [Fig. 2(c)]. The reflection coefficient of one grating cell is then:

$$R = r_1 \exp[i(2\pi d/\lambda)] + r_2 \exp[-i(2\pi d/\lambda)] = i(r_1 - r_2) \quad (1)$$

where we assumed $d = \Lambda/2 = \lambda/4$, and that $|r_1|$ and $|r_2|$ are small so that multiple reflections can be ignored. RR^* represents the energy transferred from the forward wave to the backward wave over a length Λ. Thus the coupling coefficient is:

$$\mathrm{X} = R/\Lambda = i(r_1 - r_2)/\Lambda = 2i(r_1 - r_2)/\lambda. \quad (2)$$